Lesson 1: Group Ranking methods

The problem with establishing a group ranking method is not without controversy. There is seldom complete agreement on the correct way to achieve group ranking.

I. Group Ranking Methods

A. Borda Method: assigns points based on 1st, 2nd, 3rd choice of individuals preference and then gives a point total.

Steps:

1. Assign "n" points to 1st place, n-1 points to 2nd place, ... and 1 point to last place.
2. Total each choices points.
3. The winner is the choice with the most points.

ex:

A	B	C	D
B	C	B	B
C	D	D	C
D	A	A	A
8	5	6	7

A: 8(4) + 5(1) + 6(1) + 7(1) = 50

B:

C:

D:

How do you feel about ranking this way? Is it fair? Why or why not?

B. Plurality: one way to select "the winner" is to choose the preference that is ranked first the most often.

ex:

A	B	C	D
B	C	B	B
C	D	D	C
D	A	A	A
8	5	6	7

Who wins?

A is called the Plurality Winner. Plurality Winner is based on 1st place ranking only. Was A first on the majority of ballots?

If a plurality winner is the 1st preference on over half of the ballots, then that preference is called the Majority Winner.

How do these results compare to the Borda Method?

How do you feel about always selecting the preference with the most 1st place votes?

C. Runoff Method - Many elections require a majority winner. If there is no majority winner, then a run-off election is held between the top 2 candidates.

What are some problems or issues with this type of election?

Steps to conduct a runoff:

1. Determine the number of 1st place votes for each preference.
2. Eliminate all except for the two with the highest totals.
3. Re-consider teh preference schedules that have eliminated choice ranked as #1. Of the remaining 2 choices whichever is higher gets those votes.
4. Total the votes to get the run-off winner.

ex:

A	B	C	D
B	C	B	B
C	D	D	C
D	A	A	A
8	5	6	7

How do you feel about this method? How does it compare to the Borda and Plurality?

D. Sequential Run-Off: Some elections like where the Olympic Games are held are conducted with a variation of the Run-Off method that only eliminates one choice at a time.
To conduct a sequential run-off:
1. Determine the number of 1st place votes.
2. Eliminate the choice with the smallest number of 1st place votes.
3. Transfer those votes to the highest ranked choice in that preference schedule.
4. Re-total the 1st place votes and repeat the elimination process until only 1 winner is left.

ex:

A	B	C	D
B	C	B	B
C	D	D	C
D	A	A	A
8	5	6	7

How do we feel about this method?
When would each method be good?

Go back to the food data that we ranked ealier. Find the Borda, Plurality, Run-Off and Sequential Run-Off winner for the data.

Note: On Mimeo 1 figure 1.1 is the chart we used in class.

Mimeo 1 answers
5. a)

Choice	1st	Last
A	30.8%	69.2%
B	19.2%	0%
c	23.1%	0%
d	26.9%	30.8%

6. A) 37%
B) Slightly more than a third
7. Plurality: B
Borda: D B A C
Runoff: C
Seq. Run off C
9. a) 3 doe 1st, 2 for 2nd and 1 for 3rd
b) yes - ranking would be Woodson, Manning, Leaf, Moss, Dwight, Williams, Ennis and Zerouoe (tie), McNown and Couch (tie)

So we have seen that different voting methods can lead to very different results, some of which are undesirable and make little sense. This fact led Marquis de Condorcet to offer the proposition that: "Any choice that can beat each of the others in a one-on -one race should win."

E. Condorcet Method:
Proposition: Any choice that can beat each of the others in a one-on -one race should win.

Condorcet compares every choice to every other choice:

ex:

A	B	C	D
B	C	B	B
C	D	D	C
D	A	A	A
8	5	6	7

Begin with A vs. B: A ranks higher that B on 8
A ranks lower than B on 18
Therefore A does not beat B in a one-on-one race and can't be the winner. No need to compare it to C and D.

B: We already know it beats A, so check C and D.

How do you feel about the Condorcet winner? Does it seem to be ideal? Are their any problems with it?

A	B	C
B	C	A
C	A	B
20	20	20

A	B	C
B	C	A
C	A	B
20	20	20

Let's look closer at the data:
Notice that A beats B
and that B beats C.
Mathematically what does this tell you should be true?
Why?
Is this the case?

So we have seen that group-ranking methods may violate the Transitive Property.
Because it seems contrary to your intuition, this is called a paradox.

Condorcet Paradox: This method may violate the Transitive Property.

F. Pairwise Voting: A system where 2 choices are selected and a vote is taken. The loser is eliminated, and the winner is paired against a new choice. The process continues until there is a winner.

A	B	C
B	C	A
C	A	B
20	20	20

II. Problems with Voting Methods

A. Borda

B. Plurality

C. Run-Off

D. Sequential Run Off

E. Condorcet

F. Pairwise

What do you think insincere voting is? When might it happen?

Mimeo 2

4. A with 242 pts
   B with 238 pts

7. a) 69.2% rank A last
   b) C is run off winner
      C is next to last
   c) C is seq. run off winner

8. a) A
   b) C

9. a) A, B, C, D
   b) B, A, D

Lesson 3: Arrow's Conditions and voting Approval

10 representatives of the foreign language clubs are meeting at a to select a location for their annual joint dinner. They must decide between French, German, Mexican and Italian Restaurants.
Before they vote their preferences are the following:

Mexican	French	Italian
Italian	German	French
French	Mexican	German
German	Italian	Mexican
4	3	3

One Rep, Chloe suggests that because the last two dinners were held at Mexican and German restaurants, this years dinner should be at either Italian or French. The reps agree and vote:

French - 3 Italian - 7

Another Rep, Austin, does not like Italian food. He reminds the group that a new Mexican restaurant just opened and has gotten great reviews. He thinks they should consider it. The reps agree and vote:

Italian - 3 Mexican - 7

Leah's father owns a German restaurant. She says they can get a big discount if they go there. They vote again:

Mexican - 4 German - 6

Therefore the German restaurant wins!

Look at the original preferences of the group.

Mexican	French	Italian
Italian	German	French
French	Mexican	German
German	Italian	Mexican
4	3	3

What did people feel about German food originally?

Paradoxes like these, along with insincere voting and unfair results have caused some people to look for better ways to make group ranking decisions.
Kenneth Arrow, a US Economist, formulated a list of 5 conditions that he considered necessary for fair group ranking.

1. Nondictatorship: The preferences of a single individual should not become the group ranking without considered the preferences of others.

2. Individual Sovereignty: Each individual should be allowed to order the choices in any way and to indicate a tie.

3. Unanimity: If every individual prefers one choice over another, then the ranking should do the same.

4. Freedom from Irrelevant Alternatives: The winning choice should still win if one of the other choices is removed. (the removed choice is known as the irrelevant alternative.)

5. Uniqueness of Group Ranking: The method of producing the group ranking should give the same result whenever it is applied to a given set of preferences. The group ranking should also be transitive.

Lesson 4: Weighted Voting and Voting Power

Thus far we have considered voting situations where the voters are all considered equals.
In some situations, however, the vote of some voters carries more weight than others.
Can you think of an example where this would happen?

I. Weighted Voting:
A. Def: Weighted voting occurs when a voter has more votes than another voter.
Yesterday we discussed fairness. Is it ever fair to have one person have more votes than another?

Consider the following:
Student Council for a High School has a leadership team that has a single member from each class.
Each of the reps are given votes based on the number of people that they represent.
Freshman - 2 Soph - 3 Junior - 3 Senior - 4

Any issue that comes before the board needs a majority to pass. How many would that be?

Freshman - 2 Soph - 3 Junior - 3 Senior - 4

Lets look at the possible combinations:
No members

1 member: {Fr} = 2 {So} = 3 {Jun} = 3 {Sen} = 4

2 members {F, So} = 5 {F, J} = 5 {F, Sen} = 6 {So, J} = 6 {So, Sen} = 7 {Jun, Sen} = 7

3 members {F, So, J} = 8 {F, So, Sen} = 9 {F, J, Sen} = 9 {So, J, Sen} = 10

All Members {F, So, Jun, Sen} = 12

Determine the number of ways that voting can occur we use the formula 2^n when there are n voters.

B. Coalitions: each of these collections of voters is called a coalition. Those with enough votes to pass are known as a winning coalition.

Look at our winning coalitions:

{F, So, J, Sen} = 12 Are any of the member essential to the vote?

{F, So, J} = 8 Are any of these members essential to the vote?

John Banzhaf (Law professor at GWU) reasoned that you only have power if your vote it essential to the coalition.

C. Voting Power:

1. Power Index: Measured by the # of winning coalitions to which you are essential.

Power Indexes:
Freshman - 1 Soph - 3 Jun - 3 Sen - 5

The Seniors can be expected to cast the key vote on an issue many more times than all others: 5/12

Do the Indexes reflect the original distribution of votes?

Freshman - 2 Soph - 3 Junior - 3 Senior - 4

How do you feel about the fairness of this method?

Suppose the number of votes needed to pass increased to 8 instead of a simple majority you now need a 2/3 vote. (As we just saw with the cardinals electing a new Pope.)

1 member: {Fr} = 2 {So} = 3 {Jun} = 3 {Sen} = 4

2 members {F, So} = 5 {F, J} = 5 {F, Sen} = 6 {So, J} = 6 {So, Sen} = 7 {Jun, Sen} = 7

3 members {F, So, J} = 8 {F, So, Sen} = 9 {F, J, Sen} = 9 {So, J, Sen} = 10

All Members {F, So, Jun, Sen} = 12

What are the new power indexes?

This creates another paradox. Although the votes were distributed to give greater power to the seniors, they all have the same power index.

What happens if some groups form coalitions with each other? Does this happen in our voting system in the US?