Intro To College Math

Unit 1 Logic: Purpose - To learn how to think and reason, both in mathematical and everyday contexts.

Proof that 1 + 1 = 1
Let x = 1 and y = 1, then:

Statement	Reason
1. $y = x$	
2. $-y^2 = -xy$	
3. $x^2 - y^2 = x^2 - xy$	
4. $(x + y)(x - y) = x(x - y)$	
5. $x + y = x$	
6. $1 + 1 = 1$	

Use Algebraic Properties to Supply the reasons and see if you can find the error.

(ex. Add of Equality, distributive, substitution etc.)

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To learn to think logically we need to start with the study of statements.

Statements:	Non- Statements
1 + 1 = 2.	x + 1 is an integer.
1 + 1 = 1.	x + 2 = 5.
Today is Wednesday.	HHS is the best high school!
It is August 27th.	Ms. Stachowicz is more sarcastic than any other teacher at HHS.

What makes a statement a statement?

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Definition of Statement:
A sentence with a definite truth value.

What do we mean when we say a definite truth value? How can we amend our definition to be more user friendly?

So why are these non-statements? Can we do anything to them to make them statements?

Non- Statements

x + 1 is an integer.
x + 2 = 5.
HHS is the best high school!
Ms. Stachowicz is more sarcastic than any other teacher at HHS.

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Special Types of Statements:

1. Universal Statement:

x + 1 is an integer. Why is this not a statement? What can we add to it to make it a statement?

Definition: A universal statement is a statement asserting that a certain property holds true for all elements in the set. (in other words it works all the time)

Symbol: "for all" is represented by $\forall$

$\forall$ integers , x + 1 is an integer.

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Universal Statements are powerful because they are always true for every element in a set.

Geometric Examples:
Every Square is a rectangle.

Algebraic Examples:
The Properties (be careful some need "for all" statements)

Addition Property of Equality:

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So why does this proof not work?

Let x = 1 and y = 1, then:

Statement	Reason
1. $y = x$	
2. $-y^2 = -xy$	
3. $x^2 - y^2 = x^2 - xy$	
4. $(x + y)(x - y) = x(x - y)$	
5. $x + y = x$	
6. $1 + 1 = 1$	

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Division Property of Equality:
reals a, b, c, If a = b then a/c = b/c.

Is this a Universal Statement?

How can we make it Universal?

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Truth Value:

a. True - if the statement is true for all elements of the set.

b. False - If one or more counterexamples exist

Are the following True or False:

ℝ x, $x^2 > x$

R x, Sin x < 0

All Rectangles are Squares.

All Squares are Rombi.

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Existential Statement:
An existential statement is a statement asserting that there is at least one element of a set for which a certain property holds true.

Symbol: "there exists"

Earlier we said that x+2 = 5 was not a statement
What if we change it to be:
x $\mathbb{R}$ such that x + 2 = 5.

Is it now a statement?

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Truth Value for Existential Statements:

a. True - If true for at least one element of the set.

b. False - If false for all elements of the set.

How does this relate back to the truth values for a Universal Statement?

What are the truth values for each of the following?

1. $\mathbb{R}$ x such that $x^2 > x$
2. $\mathbb{R}$ x such that $x^2 < x$
3. an integer n such that $n^2 = 2$

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Compound Statements:
Contain both and .

In English there are many ways to say the same thing, so you don't always see the phrases "For All" or "There Exists". Can you think of other ways to say these phrases?

For Every	There is
Of Any	For some

Which category (Universal or Existential) do the following fit into?
If x/3 > 5, then x > 15. If a Δ is right, then it is isosceles.

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Bellwork:

1. If t $\mathbb{R}$, which of the following statements are true?
   a) t, cos t = 0

   b) t such that cos t = 0

Be prepared to support your answer!

2. Consider the statement:
   x $\mathbb{R}$, $\sqrt{x^2} = x$.
   a) Determine the truth value. If False give a counter example.
   b) If it is false, what condition could be added to make it true?

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HW 1 Answers:

1. yes
2. yes
3. no
4. b
5. ∃
6. ∀
7. neither
8. $x = \frac{1}{2}$
9. a) yes
   b) no
   c) 1, 0 and ≥ 5
10. All students in this math class are not freshman.
11. Some students in this math class are girls.
12. False, a=0
13. False, $\log(\frac{1}{2}) = -.301$
14. x=0

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Lesson 2: Negations

How did we define what a statement was yesterday?

Every statement has a NEGATION which is also a statement.

Negations:
The negation of a statement p is a statement called not p, that, if true, exactly expresses what it would mean for p to be false.
(i.e. the denial of statement p)

Symbol: ~p

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Examples:

p: 1 + 1 = 2

~p:

p: Today is Wednesday.

~p:

You can always insert the phrase "it is not the case that________"

What would the truth values be of a Negation?

p	~p
t	
f	

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Example:

p: All teenagers have jobs.
- What type of statement is this and how do you know?
- What is the truth value? Give Support.
- Write ~p. (hint: It is not "No teenagers have jobs.")
- What is the truth value of ~p?
- What type of statement is ~p?

Negations of Universals are always Existentials!

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Special types of Negations:

1. Universal Statements: Let S be a set and p(x) be a property that may or may not be true for elements x S. Then if:
p: x S, p(x)
~p: x S such that not p(x)

Ex: Write the negation of:
p: All prime numbers are odd.
Which is true the statement or the negation?

p: All composite numbers are greater than 3.

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So if the negation of a universal statement is an existential statement, what do you think is the negation of an existential statement?

ex:
q: x ℝ such that |x| = -3.

~q:

2. Existential Statements: Let S be a set and p(x) be a property that may or may not be true for elements x S. Then if:

p: x S, such that p(x)

~p: x S, not p(x)

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Write the negation of the following.

p: Some triangles are isosceles.

q: Some rectangles are squares.

Which is true the statement or the negation?

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Compound Statements:

p: x ℝ, y ℝ such that xy = 1.

- What property is this derived from?
- Write ~p:

- Which is true and support your answer.

q: Everyone trusts someone.

Convert p into symbols (and)

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Bellwork

1. Given p: a positive real # x such that $\log_{10}x = 0$.
 a) write ~p:
 b) Which is true? p or ~p?

2. Multiple Choice: Identify the negation of
 integers 'n', integers 'a' and 'b' such that n = a/b.

A) integers n, integers a and b such that n ≠ a/b.

B) integer n, such that integers a and b, n = a/b.
C) integers n, integers a and b such that n = a/b.
D) integer n, such that integers a and b, n ≠ a/b.

Which statement is true? Original or the negation?

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HW 2 Answers

1. a person who can not drive a car.
2. a fraction that is not rational.
3. C
4. a) x ℝ such that 2x + 4≤ 0.
 b) negation
5. a) a man who is not mortal.
 b) the statement
6. C
7. a) n S such that n≥11
 b) negation
8. a) even integers m,x S.
 b) negation
9. Division by zero.
10. a, b, c, true d, e false
11. e

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Lesson 3: And / Or Statements and DeMorgan's Law

In English we have connectors that we commonly use to combine statements:
 and or not either... or neither... nor

Excerpt from the Schedule SE (1040) form from the 1998 Federal Income Tax Form.
Generally, you may use this part only if:
A - Your <u>gross</u> farm income was not more than $2400 <u>or</u>
B - Your <u>gross</u> farm income was more than $2400 <u>and</u> your net farm profits were less than $1600 <u>or</u>
C - Your <u>net</u> non-farm profits were less than $1600 and also less than 2/3 of your <u>gross</u> farm income.

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To put into simplier terms (sort of) use letters to represent statements
A - Your <u>gross</u> farm income was not more than $2400 <u>or</u>
B - Your <u>gross</u> farm income was more than $2400 <u>and</u> your net farm profits were less than $1600 <u>or</u>
C - Your <u>net</u> non-farm profits were less than $1600 and also less than 2/3 of your <u>gross</u> farm income.

f: your gross farm income was more than $2400.

p: Your net farm profits were less than $1600.

n: Your net non-farm profits were less than $1600.

g: Your net non-farm profits were less than 2/3 of your gross profit.

((not f) or (f and p) or (n and g))

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Connectors: 3 most common

	Not	And	Or
Symbol	~		
Truth Values	~p	p q	p q

p: I had Coffee

q: I had a donut

When is p q true?

When is p q true?

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Truth Tables:

p	q	p q

p	q	p q

Negations: What does it mean if an AND statement is negated?

~(p q): It is not true that I have coffee and a donut.

We don't talk like this, so how can we change the statement and have it still have the same meaning?

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p	q	p q	~(p q)	~p	~q	~p ~q

What do we notice? When 2 statements have the exact same truth values, we call them "Logically Equivalent" statements. Symbol: ≡

DeMorgans Law: ~(p q) ≡ ~p ~q

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Showing Logical Equivalence with a truth table is a method of proof.

Use a truth table to see if ~(p ~q) ≡ ~p q

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DeMorgan's Law states that negating and AND statement makes it an OR statement. What do you think Demorgan's Law states about negating an OR statement? Prove it!

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Legal speeds, L (in mph), on a particular stretch of highway are those for which $45 \le L \le 65$. Use Demorgans Laws to describe the illegal speeds.

Legal Speeds: $45 \le L$ and $L \le 65$

Illigal Speeds: $\sim(45 \le L$ and $L \le 65)$

Demorgan's: $\sim(45 \le L)$ or $\sim(L \le 65)$

Simplify: $45 > L$ or $L > 65$

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We focused on the 3 most common connectors, but what about:

either ... or   neither ... nor

means: (p q) and ~(p q)   ~p ~q

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Bellwork:

1. True of false? then explain.
integers n, an integer m such that n = 2m.

2. Given:
ℝ x and y, $x^2 + y^2 > 0$.
a) write the negation.
b) which is true the negation or the statement?

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HW 3 Answers:

1a)

p	q	p q	p (p q)
t	t	t	t
t	f	f	t
f	t	f	f
f	f	f	f

2. p≡ p (p q)
3. I don't want OJ and I don't want grapefruit with my breakfast.
4. x >5 or x ≤ 7
5. x > 5 and x ≤ 11 or 5 < x ≤ 11
6. x > 7 and x ≤ 11 or 7<x≤11
7. C
8. last column f, t, t, f
9. not rectangular or are less than 3.5 in high or less than 5 in long.
10. a) false b) true

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Lesson 4: If ... then Statements

If, then statements are found everywhere, both in side and outside of math.

If...Then statements are present whenever one statement is supposed to follow from another.

In math If ... then statements are used to form the basis of deduction and proof.

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If a quadrilateral is a rectangle, then the diagonals are congruent.

If $x > 8$, then $x^2 > 64$

If you knew Peggy Sue, then you know why I feel blue.

If you want to make the world a better place, then take a look at yourself and make that change.

Jan 16-9:06 AM

If ... then.... Statements

Called the Conditional Statement

If p, then q. symbol: $p \Rightarrow q$

If part: Hypothesis or Antecedent

Then part: Conclusion or Consequent

If a quadrilateral is a rectangle, then the diagonals are congruent.

What is the Antecedent and what is the Consequent?

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Truth Values for Conditional Statements:

* remember each p and q can be true or false

If $x > 8$, then $x^2 > 64$.

-If p is true, then q has to be true.
$x = 9$, then $x^2 = 81$ and $81 > 64$.

- If p is false, then what do we know about q?
$x = 4$, then $x^2 = 16$ but 16 is not > 64, so q is false.

$x = -9$, then $x^2 = 81$ and $81 > 64$

In a true conditional, it is possible to have the following truth values for p and q.

p	q

We are missing the "t f" row. Is it possible to have a true p and a false q?

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The only combination or truth values that a true conditional can not have is a true hypothesis and a false conclusion.

Summarize:
The conditional statement is false when p is true and q is false
The conditional statement is true in all other cases.

Truth Table:

p	q	p q
t	t	
t	f	
f	t	
f	f	

ex: Your teacher makes the following promise at the beginning of the semester: If you make an A on every test, then you will get an A in this course. You make the following test grades: A, A, B, A and your teacher gives you an A. Did your teacher keep the promise?

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Negation:
When would the negation of a conditional statement be true?

so...... $\sim(p \Rightarrow q)$ really means that $p \wedge \sim q$.

The negation of a conditional is NOT another conditional!

ex. If a student takes Math 2, then they are a Freshman.

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Universal Conditional:
If $x > 8$, then $x^2 > 64$.
What we are really saying is that: x ℝ, If $x > 8$, then $x^2 > 64$.

Negation of a Universal: x ℝ, If $x > 8$, then $x^2 > 64$.

x ℝ such that $x > 8$ and x^2 is not > 64.

Summarize: x, If p(x) then q(x).
negation: x, such that p(x) and not q(x).

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Ex. x, y R, if x < y, then |x| < |y|.

a) Write the negation.

b) Which is true, the statement or the negation? support your answer.

Jan 16-10:00 AM

Ex. y R, if a triangle has 3 congruent sides, then it is equilateral.

a) Write the negation.

b) Which is true, the statement or the negation? support your answer.

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Special Conditionals:

Inverse

Converse

Contrapositive

One of these is logically equivalent to the conditional. THe other 2 are not. Use the following statement to figure out which one has the same meaning.

If a car is a Mustang, then it is a Ford. true conditional

Jan 16-10:01 AM

Can we use a truth table to verify what we think is true from our example?

p	q	~p	~q	p q	~p ~q	q p	~q ~p
t	t						
t	f						
f	t						
f	f						

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Biconditional: p if and only if q
This means that both the conditional and the converse are true

(p q) (q p)

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Bellwork

If you study for the test, then you will do well.

Write the inverse, contrapositive and the converse. Then state when the above statement would be false.

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HW 4
1. a) hyp: $x>1$ Con: $x^2 + 3x^3 > 1$ b) true 2. b 3. false
4. true 5. Inverse
6. a) If m = 0, then the graph of y=mx+b is not an oblique line.
b) true
7. a) If a quad does not have 2 angles of = measure, then the quad does not have 2 = sides. b) false
8. Converse: If it rains tomorrow, then it will rain today.
Inverse: If it does not rain today, then it will not rain tomorrow.
9. If 2 supp angles are congruent, then they are right angles.
If 2 supp angles are right angles, then they are congruent.
10. a) false b) false
11. yes, no, yes, no
12. If one is convicted of a felony, then they are not allowed to vote.
13. If one can, then one does.
14. If Jon was not at the scene of the crime, then Jon did not commit a crime.

Jan 16-10:11 AM

Valid Arguements:
A major reason that we study logic is to
-Learn to make correct inferences or deductions.
- Be able to determine when others have made correct/incorrect deductions.

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You are about to leave for school in the morning and discover you don't have your glasses. You know the following statements are true:

- If my glasses are on the kitchen table, then I saw them at breakfast.
- If I was not reading the newspaper in the living room, then I was reading the newspaper in the kitchen.
- If I was reading the newspaper in the living room, then my glasses are on the coffee table.
- I did not see my glasses at breakfast.
- If I was reading my book in bed, then my glasses are on the bed table.
- If I was reading the newspaper in the kitchen, then my glasses are on the kitchen table.

Where are my glasses?

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I. Arguments:

A. An argument is a sequence of statements.

B. All statements except the final one are called premises or assumptions.

C. The final statement is the conclusion (or therefore before it).

Ex1.

If Jane solved the problem correctly, then Jane got 10.

Jane solved the problem correctly.

Jane got 10.

Identify the premises and the conclusion. Is the conclusion valid?

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II. Valid Arguments

A. An argument is valid iff when all the premises are true, then the conclusion is true.

B. Valid Forms of Argument

1. Law of Detachment

p   q

p

q

If the country has over 200 million people, then it imports more than it exports.

Japan has over 200 million people.

____

Do you think this valid conclusion is a true conclusion?

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Remember a valid conclusion should never be confused with a true conclusion!

You always need to check to see if the premises are true before stating the truth value of your conclusion!

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Ex3.

polygons x, if x is a hexagon, then the sum of the interior angles of x is 720°.

Polygon c has an angle sum of 540°.

____

2. Law of Indirect Reasoning.

p q

~q

~p

This is just like using the contrapositive. Remember it is equivalent to the conditional!

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Ex4.

If Mary is sick, then she has a fever.

Mary does not have a fever.

____

Ex5.

If a figure is a square, then it is a parallelogram.

If a figure is a parallelogram, then the diagonals bisect each other.

____

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Example 5 is the

3. Law of Transitivity

p q

q r

p r

Use these Laws to go back to the original problem and find my glasses!

Jan 16-3:51 PM

You are about to leave for school in the morning and discover you don't have your glasses. You know the following statements are true:

- If my glasses are on the kitchen table, then I saw them at breakfast.
- If I was not reading the newspaper in the living room, then I was reading the newspaper in the kitchen.
- If I was reading the newspaper in the living room, then my glasses are on the coffee table.
- I did not see my glasses at breakfast.
- If I was reading my book in bed, then my glasses are on the bed table.
- If I was reading the newspaper in the kitchen, then my glasses are on the kitchen table.

Where are my glasses?

Jan 16-3:52 PM

Bellwork

1. Given the statement: "$\exists x \in \mathbb{R}$ such that if $x = |x|$, then x = x.
   - a. Write the negation.
   - b. Which is true? The statement or its negation?
2. If it is yellow, then it is a banana.
   A bus is yellow.
   ∴ A bus is a banana.
   - a. Is the conclusion **valid**?
   - b. Is the conclusion **true**?
3. Write the inverse, converse and Contrapositive of the following:
   If you read books, then you are bad at math.
4. What would make the statement in #3 false?

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HW 5 answers

1. a. 1&2 b. 3 c. conditional - law detachment d. no e. yes
2. valid, Law of indirect reasoning
3. Valid, Law of Transitivity
4. Valid, Law of Detachment
5. a) Mary is not at home. b) p q c) Law of Indirect Reasoning
   ~q p
6. b) yes Law of Transitivity
7. Diagonals of ABCD bisect each other.
8. Indirect Reasoning; Yes
9. If it involves a quad somersault, then it is not attempted.
10. -3 and -1 are not positive $\mathbb{R}$
11. If a fruit does not come from New Zealand, then it is not a kiwi.
12. a) A b)D
13. a) y R such that $y^2+3 < 3$ b) the statement
14. True

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Lesson 6: Invalid Arguments

Consider the following argument:
If a person is a member of the band, then the person plays an instrument.
Natalie Plays an instrument.
Natalie is a member of the band.

Do you think the conclusion is valid?

It is possible for both premises to be true and the conclusion to be false.

Jan 17-10:06 AM

I. Invalid Arguments

A. An invalid Argument: When all the premises are true but the conclusion could be false. (in some cases it can also be true, but if there is a possibility of false, then it is an invalid argument.)

B. Forms of Invalid Arguments.

1. Converse Error p q
   q
   p

A person cleared of the crime argues the following:
If a person has not done anything wrong, he is cleared at the end of the investigation.
I was cleared at the end of the investigation.
I have not done anything wrong.

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2. Inverse Error p q
~p
~q

If a person is a member of the Spanish Club, then a person speaks Spanish.
William is not a member of the Spanish Club.
William does not speak Spanish.

Again, remember that the conclusion can be true but is not guaranteed, therefore the argument is invalid.

Jan 17-10:26 AM

Determine whether the following are valid or Invalid. Write the form of the argument to support your answer.

1. If a number is divisible by 8, then it is divisible by 4.
20 is not divisible by 8.
20 is not divisible by 4.

2. If a number is divisible by 8, then it is divisible by 4.
30 is not divisible by 4.
30 is not divisible by 8.

3. If a person is caught driving under the influence, then the person's license is suspended.
Jim was caught driving under the influence.
Jim's license was suspended.

4. If it is Sunday, then Nate reads the comics.
Nate reads the comics.
It is Sunday.

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Read the first two statements and determine whether the 3rd statement would be a correct conclusion if the first two statements are true. If so, write 'valid'. If not, tell if you have a converse error or inverse error.

If lightning occurs, then the soccer game will be canceled.
An official sees lightning and hears thunder.
Therefore, the soccer game is canceled.

If you forgot your pencil, you may borrow one of mine.
Anna forgot her pencil.
Therefore, she may borrow one of mine.

All equilateral triangles are isosceles.
The triangle is not equilateral.
Therefore, the triangle is not isosceles.

All carbonated soft drinks have bubbles.
You drink something that is bubbly.
Therefore, it is a carbonated soft drink.

Jan 27-12:37 PM

- When I work a logic example without grumbling, you may be sure it is one I can understand.
- This example is not arranged in regular order like the ones I am used to.
- No easy examples give me a headache.
- I can't understand an example that is not arranged in regular order like the ones I am used to.
- I grumble when I work an example only if I get a headache.

Suppose each of the above sentences is true. Must the following conclusion also be true?

This example is not easy.

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Use the transitive property to put these statements in order:

____If the mud runs into the river, then the gills of the fish get clogged w/ silt.

____If there is a forest fire, then there is nothing to trap the rain.

____ If the gills get clogged w/ silt, then fish can't breathe.

____If you are careless with fire, then there is a forest fire.

____If fish can't breathe, then they will die.

____Therefore, if you are careless with fire, then fish will die.

____If there is nothing to trap the rain, then mud runs into the river.

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____If you drink while you drive, then your reflexes are not good.

____If you have severe injuries, then you may die.

____If you do not have an accident, then you can react quickly.

____If you have an accident, then you may have severe injuries.

____If you react quickly, then your reflexes are good.

Therefore, ____

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HW 6 Answers:

1. a) Inverse b) Invalid
2. a) Contrapositive b) Valid
3. a) Converse Error b) Invalid
4. a) Statement b) Valid
5. a) Inverse Error b) Invalid
6. a) Converse Error b) Invalid
7. a) Converse Error b) Invalid
8. a) Contrapositve b) valid
9. a) Converse Error b) invalid

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Write the Negation:

1. If you behave, then we will go outside.

Is the argument valid? Why or why not?

If you practice, then you will get better.

Austin got better.

Therefore, Austin practiced.

Jan 22-7:57 AM

review answers: 1. D 2. C 3. True 4. D and E 5. True

6. $\exists \mathrm{Re}\, alNumbers\ y\ such\ that\ 0 + y \neq y$

7. x $\geq$ -8 and x < 12

8. The bald eagle is not the national bird OR the Star Spangled Banner is not the national anthem.

9. If the person is admitted to an R movie, then the person is at least 17.

10. s $\leq$ 4 11. C 12. B

13. If 2 lines are // then the corr. angles are congruent. AND If the corr. angles are congruent, then the lines are //.

14. indirect reasoning 15. Inverse Error 16. Valid Transitive 18. False

Feb 3-7:13 AM