**Logic HW 1**

In 1-3, tell whether the sentence is a statement.

There is exactly one solution to the equation 2x=5.

The equation has three solutions.

Multiple Choice. Which is the true statement?

In 5-7, identify the sentence as a universal statement, existential statement, or neither.

There exists a bird b such that b cannot fly.

For all real rivers r in North America, r flows into the Mississippi.

Chicago is the capital of Illinois.

Find a counterexample to show that the following statement is false:

Let p(x) be

* 1. Is p(x) true when x = 5?
  2. Is p(-1) true?
  3. For what integer values of x is p(x) true? (You do not have to prove your answer is correct.)

Identify a property that is true for all students in your math class. Write this as a universal statement.

Identify a property that is true for some, but not all, students in your math class. Write this property as an existential statement.

True or False? If false, find a counterexample.

True or False? Justify your answer.

Find a counterexample to show that the following statement is false:

For all real numbers x there exists a real number y such that

**Logic HW 2**

**In 1 and 2, write a negation for the following statements:**

1. Every person can drive a car. 2. All fractions are rational numbers.

3. Multiple Choice. The negation of: Some quadratic equations have three solutions.

* 1. Some quadratic equations do not have three solutions.
  2. All quadratic equations have three solutions.
  3. No quadratic equations have three solutions.
  4. There exists a quadratic equation with three solutions.

**In 4 and 5, a statement is given.**

1. **Write the negation of the statement**
2. **Which is true: the given or its negation?**

4. p: For all real numbers x, 2x + 4 > 0.

5. q: All men are mortal. (Socrates)

*6. Multiple Choice*. Give the negation of: All people under 21 can legally buy alcohol in Illinois.

a. All people under 21 cannot legally buy alcohol in Illinois.

1. All people under 21 can legally buy alcohol in Illinois.
2. Some people under 21 cannot legally buy alcohol in Illinois.
3. Some people under 21 can legally buy alcohol in Illinois.
4. No one under 21 can legally buy alcohol in Illinois.

**In 7 and 8, S = {1, 3, 5, 7, 9, 11}**

1. **Write the negation of the statement**
2. **Which is true: the given or its negation?**

7. r: For all n in S, n < 11

8. q: There exists an even integer m such that m is in S.

**9. Find the flaw in the following argument.**

Let x = 2 and y = 2.

Then

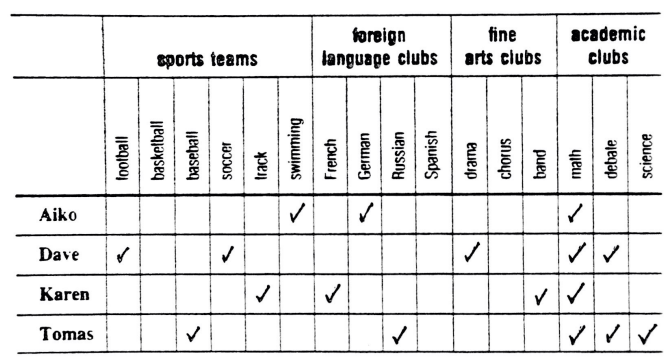
So

So

So

But because

**Refer to the information in the table below.** At King High School, extracurricular activities are offered in four categories: sports teams, foreign language clubs, fine arts clubs, and academic clubs.



10. Determine whether each of the following statements is true or false. Justify your answer.

1. Some foreign language club has no members from the sample.
2. *Multiple Choice.* You learned in geometry that the measure of an angle inscribed in a semicircle is . Which of the following statements is not equivalent to the one above?
3. All angles which are inscribed in a semicircle have a measure of .
4. If A is an angle inscribed in a semicircle, then mA =
5. Every angle which is inscribed in a semicircle has a measure of .
6. All of the above are equivalent.

**Logic HW 3**

1. a. Complete the truth table below.

p q pq p(pq)

1. Two of the columns should be the same in your answer to part a. What does this mean?
2. Use a truth table to prove the second of DeMorgan’s Laws:

~(p  q) (~ p)  ( ~q).

1. Use DeMorgan’s Laws to express the negation of: I want orange juice or I want grapefruit juice with my breakfast.

**In 4-6, let x be a real number. Let p(x), q(x), and r(x) be the following sentences:**

**p(x): x > 5 q(x): x > 11 r(x): x 7**

Write a sentence for the following:

1. pr 5. p ~q 6. ~ r  ~q

*7. Multiple Choice.* Which of the following is the negation of: You are driving over the speed limit and you get a ticket?

* 1. You are not driving over the speed limit and you do not get a ticket.
  2. You are driving over the speed limit and you do not get a ticket.
  3. You are driving over the speed limit or you do not get a ticket.
  4. You are driving over the speed limit or you get a ticket.

8. The symbol *xor* is sometimes used to denote “exclusive or.” That is, if p and q are statements, then p xor q is defined to be true if and only if p is true or q is true, but not both are true.

* 1. Construct a truth table for xor using the definition of its truth values given here.
  2. Use a truth table to prove that: p xor q (p q)  (~ (p q)).

9. Post office regulations used in 1990 indicated that all pieces that were to be mailed and were ¼ inch or less thick had to be rectangular in shape, at least 3 ½ inches high, and at least 5 inches long. Use DeMorgan’s Laws to describe pieces that were inch thick but could not be mailed.

10. True or false?

a. rectangles r, r is a square. b. a rectangle r such that r is a square.

**Logic HW 4**

1. Let x be a real number. Consider the statement: If x > 1, then
2. Identify the antecedent, the conclusion, the consequent, and the hypothesis.
3. Is the conditional true or false?
4. If Sandra is on the swim team, then she swims every day. Which would tell you that the statement is false?
5. Sandra is not on the swim team, and she swims every day.
6. Sandra is on the swim team, and she does not swim every day.
7. Sandra is not on the swim team, and she does not swim every day.
8. *True or False?* The negation of: Is

**Determine whether the conditional is true or false. If false, give a counterexample.**

1. If a quadrilateral is a square, then it is a parallelogram.
2. Are the following contrapositive, inverse, or converse) of each other?

**In 6 and 7: a. Write the contrapositive of the statement, and b. Determine whether or not the contrapositive is true.**

1. If the graph of y = mx + b is an oblique line, then m 0.
2. If a quadrilateral has two sides of equal length, then the quadrilateral has two angles of equal measure.
3. Write the converse and inverse of: If it rains today, then it will rain tomorrow.
4. Given the statement: Two supplementary angles are congruent if and only if they are right angles. Write two if-then conditionals contained in this statement.
5. Suppose that p(n) and q(n) are the sentences

p(n): n is a prime number q(n) n is an odd number

1. Determine the truth or falsity of
2. Determine the truth or falsity of
3. Let p(x) be: If , then
   1. Is p(7) true?
   2. Is p(.7) true?
   3. Is p(2) true?
   4. Is p(x) true for all real numbers x?

**In 12 and 13, rewrite each statement in if-then form.**

1. No one who has been convicted of a felony is allowed to vote.
2. Those who can, do.
3. Consider the statement: If Jon committed a crime, then Jon was at the scene of the crime.
   1. Write the contrapositive.
   2. What is the legal significance of the contrapositive?
4. Show by a truth table that is not logically equivalent to.
5. Show by a truth table that the converse of is logically equivalent to the inverse of .
6. Rewrite in if-then form: A satellite is in orbit only if it is at a height of at least 200 miles above the earth.

**Logic HW 5**

1. Consider the following argument:

For all integers n, if n is divisible by 3, then its square is divisible by 9.

10 is divisible by 3.

is divisible by 9.

1. Identify the premises of the argument.
2. Identify the conclusion of the argument.
3. Write the form of the argument.
4. Is the conclusion true?
5. Is the conclusion valid?

In 2-4, determine whether the argument is valid. If so, identify the valid argument form.

1. If a person is at least 16 years old, then the person is eligible for a driver’s license.

Sara is not eligible for a driver’s license.

Sara is not at least 16 years old.

1. real numbers x, if

real numbers x, if then

real numbers x, if , then

1. If Ken wants to see the world, then Ken will join the Navy.

Ken wants to see the world.

Ken will join the Navy.

1. You know that your Mary always answers the phone if she’s at home. You just called her and there was no answer.
   1. What valid conclusion can you draw?
   2. Write the form of this argument.
   3. Name the law that guarantees the validity of this argument.
2. Consider the argument:

If the drought continues, then the corn crop will be low.

If the corn crop is low, then the price of corn will go up.

So, if the drought continues, then the price of corn will go up.

1. Write the form of this argument.
2. Is the above argument valid? Explain.
3. Deduce a valid conclusion from these three true premises:

The diagonals of a parallelogram bisect each other.

All rhombuses are parallelograms.

ABCD is a rhombus.

1. Faced with the task of determining if a crown for the king of Syracuse contained anything other than gold, Archimedes (c. 287-212 B.C.) is said to have used the following principle. *If an object is made of pure gold, then its weight equals the product of its volume and the density of gold.* Archimedes devised a method to measure the volume of the crown accurately. Then he calculated that the weight was less than it would be if the crown was made of pure gold. He, therefore, concluded that the crown was not made of pure gold.
2. What form of argument was he using?
3. Is this a valid form?
4. Consider the statements in this adaptation of a Lewis Carroll puzzle.

If an acrobatic feat is possible, then it does not involve a quadruple somersault.

If an acrobatic feat is impossible, then it is not announced in the bills of a circus.

If an acrobatic feat is not announced in the bills of a circus, then it is not attempted by the circus acrobats.

Deduce a valid conclusion. (Hint: You may need to write the contrapositive of some of the statements.)

1. Find the error that leads to the false conclusion.

positive real numbers s and t, if then

1. *Multiple Choice.* Identify the correct contrapositive for: If a fruit is a kiwi, then it comes from New Zealand.
   1. If a fruit is not a kiwi, then it does not come from New Zealand.
   2. If a fruit is a kiwi, then it does not come from New Zealand.
   3. There is a fruit that is a kiwi and does not come from New Zealand.
   4. There is a fruit that is not a kiwi and does not come from New Zealand.
2. Use the choices of Question 11. Which is the inverse of the given sentence? Which is the negation?
3. Consider the statement:

real numbers y,

* 1. Write the negation of this statement.
  2. Which is true: the statement or its negation?

1. *True or False.* The negation of *There exists an integer n such that is not an integer* is *For all integers n,*  *is an integer.*

**Logic HW 6**

**In 1-5, a. identify the type of argument and b. tell whether or not it is valid.**

1. If you buy a ticket to the concert, you will go.

You don’t buy a ticket to the concert.

You don’t go.

1. If a person does not register to vote, then the person cannot vote.

John can vote.

John has registered to vote.

1. If this is a presidential race, education is a top issue.

Education is a top issue.

This is a presidential race.

1. Assume that Peter has an answering machine attached to his telephone. He turns it on whenever he is leaving home. When Sara called, she got the message on the answering machine. Sara concluded that Peter was not home.
2. Write the form of argument Sara used to draw her conclusion.
3. Is it valid or invalid? Explain.
4. Given the following argument:

* 1. Write the form of the argument. b) Is the argument valid or invalid?

1. Given the following argument:

For all persons p, if p is President of the U.S., then p is at least 35 years old.

Queen Elizabeth II is at least 35 years old.

Queen Elizabeth II is President of the U.S.

* 1. Write the form of the argument.
  2. Are the premises true? Is the conclusion true?
  3. Is the argument valid or invalid? Justify your answer.

**In 7-9, write the form of the argument and determine whether the argument is valid or invalid. If the argument is invalid, identify the type of error made in the argument.**

1. If the land is covered with ice, then the land is Antarctica. If the land is Antarctica, then there are research stations there. If the land has research stations, then scientific study is being conducted. Scientific study is being conducted on the land. Therefore, the land is covered with ice.
2. If x is a real number, then If x is an imaginary number, then 2i is an imaginary number. Therefore 2i is not a real number.
3. If you send a minimum order of $10 to a mail-order house, then your name is put on a mailing list. If your name is on the mailing list, then you will receive many catalogs in the mail. You receive many catalogs in the mail. Therefore, you must have sent a minimum order of $10 to the mail-order house.

**Logic Review**

**In 1 and 2, match each sentence with the best description of it:**

1. Statement b. Universal statement c. Existential statement d. None of these
2. There is a condor that has been born in captivity.
3. *True or False.* mammals m, m is not a mosquito.
4. Identify the statements equivalent to: *All Olympic medalists are tested for illegal drug use.*
   1. *No Olympic medalists are tested for illegal drug use.*
   2. *There exists an Olympic medalists who is tested for illegal drug use.*
   3. *Some Olympic medalist is tested for illegal drug use.*
   4. *For all Olympic medalists m, m is tested for illegal drug use.*
   5. *If an individual is an Olympic medalists, then the individual is tested for illegal drug use.*
   6. None of these sentences are equivalent to the original.
5. *True or False.*
6. Write the negation of the statement in Question 5.
7. Rewrite spelling out the implied *and* and *or.*
8. Write the negation of *The bald eagle is the national bird and “The Star-Spangled Banner” is the national anthem.*
9. Write the following statement as an if-then statement:

*A person can be admitted to an R-rated movie only if the person is at least 17 years old.*

1. Imagine that s is a particular real number. Consider the sentence: *If*

Under what circumstances is this sentence false?

1. *Multiple Choice.* A conditional is logically equivalent to which of the following?
   1. Its converse
   2. Its inverse
   3. Its contrapositive
2. Identify the sentence which is the contrapositive of: *If you travel to Africa, then you need to have a malaria shot.*
   1. *If you need to have a malaria shot*, *then you are traveling to Africa.*
   2. *If you do not need to have a malaria shot*, *then you are not traveling to Africa.*
   3. *If you are not traveling to Africa, then you do not need to have a malaria shot.*
3. Write the following theorem in an equivalent way using two *if-then* statements:

*Two lines are parallel if and only if, when cut by a transversal, corresponding angles have the same measure.*

**In 14 and 15, match the argument with the appropriate argument form(Law of Detachment, Law of Indirect Reasoning, Law of Transitivity, Converse Error, Inverse Error) and indicate if the argument is valid or invalid.**

**Determine whether the argument is valid or invalid. Justify your reasoning.**

1. *If Pete hit three home runs in the game, then his team won. If Pete was not named most valuable player, then his team did not win.*

*Pete hit three home runs in the game.*

*Therefore, Pete was names most valuable player.*

1. Write the truth table for *~(p q)*.
2. Consider the universal statement: Is the statement true or false? If true, explain why. If false, give a counterexample.