Unit 7: Matrix Applications

Review:
1. What are matrix Dimensions?

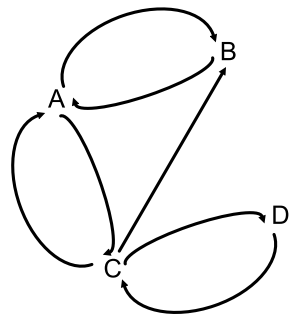
$$\begin{bmatrix} 2 & -3 & 5 \\ 0 & -1 & 3 \end{bmatrix}$$

2. When can we add matrices? Multiply?

3. What is a square matrix?

4. How do we find the inverse of a matrix?

A, B, C and D represent 4 ships at sea that are within communication range. The arrows indicate the direction of radio transmission.



I. Communication Matrix

A. Characteristics of:

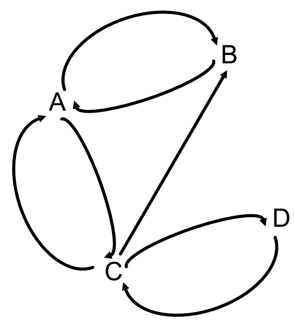
1. Square Matrix (dim = # of objects)

2. Rows represent who the communication is from
Columns represent who the communication is to

3. Entries: 1's if direct communication from one ship to the other is possible.
0's if communication is not possible.

4. 0's are on the main diagonal because we assume a ship will not send communication directly to itself.

5. Symbol: M



	P	Q	R	S	T
P	0	1	0	1	1
Q	1	0	1	1	0
R	0	1	0	0	1
S	1	0	0	0	1
T	1	0	0	0	0

Draw a communication network

B. Special Matrices:

1. M - Gives the direct communication links between objects

2. M^2 - Gives the indirect (or 2 step) communication That use another object as a relay.

$$M^2 = \begin{bmatrix} 2 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & 2 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix}$$

There are 2 ways that communication between A and itself can be done in 1 relay (2-steps)

D can communicate with A using a relay.

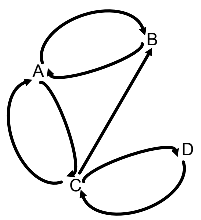
If M gives us direct communication and M^2 gives us communication with 1 relay, what does $M^2 + M$ give us?

3. $M^2 + M$ gives us the communication between objects using at most 1 relay.

$$M^2 + M = \begin{bmatrix} 2 & 2 & 1 & 1 \\ 1 & 1 & 1 & 0 \\ 2 & 2 & 2 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

There are 2 paths from C to B that take no more than 1 relay. C-B and C-A-B

Are all ships able to communicate with each other in less than 2 steps?



4. M^3 gives us 3 step communication or 2 relays.

$M^3 + M^2 + M$ = # of paths that communication can exist in at most 2 relays or 3 steps.

Lesson 2: Markov Chains

Objective: To use matrices to make predictions when we know where things currently stand and understand how they are changing.

A **Markov Chain** is a process that arises naturally in problems that involve a finite number of events or states that change over time.

Students at Lincoln High have two choices for lunch. They can either eat in the cafeteria or eat elsewhere. The director of food service is concerned about being able to predict how many students can be expected to eat in the cafeteria over the long run. She has asked the discrete mathematics class to help her out by conducting a survey of the student body during the first two weeks of school. The results of the survey show that if a student eats in the cafeteria on a given day, the probability that he or she will eat there again the next day is 70% and the probability that he or she will eat elsewhere is 30%. If a student does not eat in the cafeteria on a given day, the probability that he or she will eat in the cafeteria the next day is 40% and the probability that he or she will eat elsewhere is 60%. On Monday, 75% of the students ate in the cafeteria and 25% did not.

if a student eats in the cafeteria on a given day, the probability that he or she will eat there again the next day is 70% and the probability that he or she will eat elsewhere is 30%. If a student does not eat in the cafeteria on a given day, the probability that he or she will eat in the cafeteria the next day is 40% and the probability that he or she will eat elsewhere is 60%. On Monday, 75% of the students ate in the cafeteria and 25% did not.

The director wants to continue this process for many days. So the tree diagram will soon become impractical and too large to work with.

Therefore, we begin with a matrix called the **initial distribution** (Monday's data) of the student body. We will use a row **(initial - state)** matrix, D_0 where

$D_0 =$ [ ]

Movement from one state to another is called a **transition**, so the data about how students choose to eat from one day to the next is written in a matrix called a **transition matrix**, T, where

2. Calculate the product of matrix D_0 and matrix T: (Call the resulting matrix D_1)

5. Now, let's consider how D_2 was calculated.

6. Find the distribution on Friday. (Call the resulting matrix D_4)

7. If there are 1000 students at the school, how many will eat in the cafeteria and how many will eat elsewhere?

8. The movement of students from one state to another can also be shown with a weighted diagraph called a **transition diagraph** or **state diagraph**.

3. Compare the calculations with those made in the tree diagram model. What do you notice?

4. Find how students will eat on Wednesday. (Call the resulting matrix D_2)

$D_2 = D_1T = [\quad\quad] \begin{bmatrix} \quad & \quad \\ \quad & \quad \end{bmatrix} = [\quad\quad]$

Lesson 3: Leslie

P.H. Leslie developed a population growth model in 1945 (before computers and calculators):

His model is used to determine:

1. ____

2. ____

P.H. Leslie found by studying the female populations that,

If ____

Then ____

Terminology:

1. Birthrate: ____

2. Survival Rate: ____

3. Initial Population: _______________________________________

Equations:

1. Reproduction: ____

2. Development and Mortality: ________________________________

Age (months)	Birthrate	Survival Rate
0 3	0	0.6
3 6	0.3	0.9
6 9	0.8	0.9
9 12	0.7	0.8
12 15	0.4	0.6
15 18	0	0

Age (months)	0 3	3 6	6 9	9 12	12 15	15 18
Initial Distribution	15	9	13	5	0	0

1. Find the total initial population of rats.

2. Find the number of female rats that survive in each age group after one cycle (3 months).

3. Find the number of rats born after one cycle (3 months).

4. Write a new population distribution for the rats after one cycle (3 months).

5. What is the total population after one cycle (3 months)

6. Find the number of female rats that survive in each age group after two cycles (6 months).

7. Find the number of rats born after two cycles (6 months).

8. Write a new population distribution for the rats after two cycles (6 months).

9. What is the total population after two cycles (6 months)?

The Leslie Matrix (L): ____

Column 1: ____

Columns 2 and on: ____

Initial Matrix (P_o): ____

Sum Matrix:____

Age (months)	Birthrate	Survival Rate
0 3	0	0.6
3 6	0.3	0.9
6 9	0.8	0.9
9 12	0.7	0.8
12 15	0.4	0.6
15 18	0	0

Age (months)	0 3	3 6	6 9	9 12	12 15	15 18
Initial Distribution	15	9	13	5	0	0

1. Write the Leslie Matrix for this population.

2. Write the initial distribution matrix for this population.

3. Find the population distribution after one cycle (3 months).

4. Find the population distribution after two cycles (6 months).

5. Find the population distribution after three cycles (9 months).

6. Find a formula that can be used to find the population distribution after any number of cycles.

7. Find the population distribution after the tenth cycle.

8. Find the population distribution after the twentieth cycle.

9. Find the total population after the tenth cycle.

10. Find the total population after the twentieth cycle

11. How long will it take for the total population to reach 150?

Lesson 4: Leontief Input / Output models

I. Consumption Matrix

A. Definition: ____

B. Represents: ____

5. Transfer the dollar amounts to percents in order to apply it to any situation.

a. example: In order to produce $120 million dollars worth of auto, they needed $24 million from itself, $60 million from oil, and $36 million from transportation.

b. Consumption Matrix

Consumption (Input)

Production (Output)	A	O	T	Consumer Demand	Total
A	24	15	60	21	120
O	60	30	30	30	150
T	36	60	30	174	300

1. Rows give ____

a. example: ____

2. Columns give ____

a. example: ____

3. Consumer Demand ____

4. Total: ____

6. Use the Consumption Matrix above to answer:

a. On which sector does oil rely on the most?

b. Which sector depends the least on auto?

c. On which sector does auto depend on the most?

II. Weighted Diagram

A. Shows ____

B. Draw a weighted diagraph to represent the consumption matrix above.

Internal Usage: ____

Formula: ____

External Demand: ____

Formula: ____

Total Production: ____

Formula: ____

(Output)	A	B	C	Demand	Total
A	60	70	40	70	240
B	60	60	50	45	200
C	48	74	60	68	250

1. Write a consumption matrix for the table above.

2. Which sector is most dependent on A?

3. Which sector is least dependent on B?

4. On which sector does C depend on the most?

5. On which sector does B depend on the least?

6. On which sector does C depend on the least?

7. Draw a weighted diagraph for the consumption matrix above?

Auto, Oil, Transportation Example

1. If the company produces $50 million of Auto, $45 million of Oil, and $57 million of Transportation, how much of that is used internally?

2. Using those same amounts, how much is available for external demand?

3. If the consumer demand is $100 million of Auto, $85 million of Oil, and $120 million of Transportation, how much needs to be produced?

(output)	B	M		
B	3	8	89	100
M	1	4	95	100

1. Write a consumption matrix for this two sector economy.

2. Draw a weighted diagraph for the economy.

3. If the company produces $576 of batteries and $604 of motors, how much is used internally?

4. How much of each sector is available for external demand if the company produced $576 batteries and $604 motors.

5. If the consumer demand is $\begin{bmatrix} 450 \\ 538 \end{bmatrix}$, how much of each sector needs to be produced?

Lesson 5: Game Theory

I. Game Theory:
   A. Strategies: ____
   B. Game Theory: ______________________________________
   C. Strictly Determined Games: __________________________
   D. Payoff Matrix: _____________________________________

1. Write a consumption matrix for this economy.

(Output)	S	E	O		
S	10	40	20	30	100
E	15	10	45	30	100
O	5	25	15	55	100

2. Draw a weighted diagraph for the economy.

3. If the company produces $625 of service, $825 of Electricity, $944 of Oil, how much is used internally?

4. How much is available for external sales?

5. If the consumer demand is $\begin{bmatrix} 200 \\ 250 \\ 400 \end{bmatrix}$, how much of each sector needs to be produced?

Consider a simple coin-matching game that Steve and Tina are playing. Each conceals a penny with either heads or tails turned upward. Each conceals a penny with either heads or tails turned upward. They display their pennies simultaneously. Steve will win three pennies from Tina if both are heads. Tina will win two pennies from Steve if both are tails, and one penny from Steve if the coins don't match.

Write a matrix showing Steve's point of view.

Write a matrix showing Tina's point of view.

Consider Steve's point of view to find each person's strategy.

a. Row Minimum: ____________________
b. Maximin: ____________________
c. Steve should display? ____________________
d. Column Maximum: ____________________
e. Minimax: ____________________
f. Tina should display? ____________________
g. Saddle Point: ____________________

In a competition between two pizza restaurants, Dino's and Sal's, both are considering four strategies: running no special, offering a free mini pizza with the purchase of a large pizza, offering a free medium pizza with the purchase of a large one, and offering a free drink with any pizza purchase.

Dino:	NS	MP	Med	Drink
No Special	200	–400	–300	–600
Mini Pizza	500	100	200	600
Medium	400	–100	–200	–300
Drink	300	0	400	–200

1. Find the row Minimums
2. Maximin?
3. Dino's best strategy?
4. Column Max
5. Minimax
6. Sal's best strategy?
7. Saddle Point?