

What does it mean for a person to get a fair share?

Robin Williams

What type of objects would be easy to divide up? (**continuous case**)

What type of objects would be hard to divide up? (**discrete case**)

I. Estate Division:

	House	Boat	Car
Tom	80,000	5,000	8,000
Bob	70,000	9,000	11,000
Sue	76,000	7,000	13,000

Ex. Tom, Bob and Sue are heirs to an estate that includes a house, boat, car and \$150,000. How should they divide up the estate so that everyone gets a fair share?

Step 1: Heirs assign dollar amounts to what they value each object as being worth.

Step 2: Award the bidders – each item goes to the person who values it the most.

Step 3: Determine what each person feels is a fair share

Step 4: Divide up cash to equal their fair share.

Step 5: Divide up remaining cash.

Example 2: Kenny, Josh and Alex are heirs to an estate the contains a car, dog and piano. The following are their bids.

	Car	Dog	Piano
Kenny	2500	300	800
Josh	2000	600	700
Alex	2400	500	1000

1. Award the bidders:

2. Calculate each person's fair share:

Example 3: Four partners (Adam, Benson, Cagle, and Duncan) jointly own a piece of land which is subdivided into 4 parcels (s1, s2, s3, and s4). The following table shows the percentage of the value of the land that each parcel represents to each partner.

	s1	s2	s3	s4
Adams	30%	24%	20%	26%
Benson	35%	25%	20%	20%
Cagle	25%	15%	40%	20%
Duncan	20%	20%	20%	40%

- a) Indicate which of the four parcels are fair shares to Adams.
- b) Indicate which of the four parcels are fair shares to Benson.
- c) Indicate which of the four parcels are fair shares to Cagle.
- d) Indicate which of the four parcels are fair shares to Duncan.
- e) Assuming that the four parcels cannot be changed or further subdivided, describe a fair division of the land.

Shift	A	B	C	D
# of Patients	871	1029	610	190
Quotas				

A clinic has a nursing staff of 300 nurses working in four shifts, A, B, C, D. The number of nurses apportioned to each shift is based on the number of patients per shift, given in the following table.

Lesson 2:
Apportionment:

I. Ideal Ratio and Quotas

Ideal Ratio:

Quotas:

Discrete Town has 5 districts. District 1 has 3640 people. District 2 has 5760, District 3 has 10,300 people, District 4 has 2,500 and District 5 has 7,800. The town council has 30 seats.

State	A	B	C	D
Population (in thousands)	275	383	465	767
Quota				

The republic of Mathtown is composed of four states, A, B, C, and D. According to the country's constitution, the congress will have 30 seats, divided among the 4 states according to their respective populations. The table below shows each state's population.

II. Truncating: _____

7.56 becomes: _____

Apportion Method #1: Hamilton's Method

1. Calculate the quotas
2. Truncate each quota and assign that number to each group
3. Give the surplus items, one at a time, to the groups with the largest decimal parts until there are no more surplus items.

Examples: Use Hamilton's method for each of the following examples.

Shift	A	B	C	D
# of Patients	871	1029	610	190
Quotas				
Truncated Quotas				
Hamilton's Method				

A clinic has a nursing staff of 300 nurses working in four shifts, A, B, C, D. The number of nurses apportioned to each shift is based on the number of patients per shift, given in the following table.

Discrete Town has 5 districts. District 1 has 3640 people. District 2 has 5760, District 3 has 10,300 people, District 4 has 2,500 and District 5 has 7,800. The town council has 30 seats. Apportion the seats using Hamilton's method.

III. Jefferson's Method

Modified Divisor (d):

Modified Quota:

Steps:

1. Divide each group's population by d.
2. Round down to the nearest whole number.
3. Find the sum of the whole numbers. If the sum is the number of things to be apportioned, stop. If not, go on to step 4.
4. If the sum is too high, make d bigger and repeat steps 1 – 3. If the sum is too low, make d smaller and repeat steps 1 – 3.

The republic of Mathtown is composed of four states, A, B, C, and D. According to the country's constitution, the congress will have 30 seats, divided among the 4 states according to their respective populations. The table below shows each state's population.

State	A	B	C	D
Population (in thousands)	275	383	465	767
Quota				

An HMO has 70 doctors to be apportioned among six clinics. The HMO decides to apportion the doctors based on the average weekly patient load for each clinic

Clinic	A	B	C	D	E	F
Average Weekly Patient Load	309	591	389	683	419	479
Quota						

Lesson 3: More Apportionment

Webster Method:

1. Round quotas based on arithmetic mean (.5 and above round up, .4 and below round down). Add the resulting integers together. If the total equals the number of items to be apportioned, stop. If not, go on.
2. Divide each population by a modified divisor (d).
3. Round the resulting numbers based on the arithmetic mean. Add the resulting integers together.
4. If the total equals the number of items to be apportioned, stop. If not, repeat steps 2 and 3.

**When determining the modified divisor, use the same rule as the Jefferson Method: If the sum is too high, make d bigger, if the sum is too low, make d smaller.

1. A clinic has a nursing staff of 300 nurses working in four shifts, A, B, C, D. The number of nurses apportioned to each shift is based on the number of patients per shift, given in the following table.

Shift	A	B	C	D
# of Patients	871	1029	610	190
Quotas				

The republic of Mathtown is composed of four states, A, B, C, and D. According to the country's constitution, the congress will have 30 seats, divided among the 4 states according to their respective populations. The table below shows each state's population.

State	A	B	C	D
Population (in thousands)	275	383	465	767
Quota				

A rapid transit service operates 130 buses along six routes, A, B, C, D, E, and F. The number of buses assigned to each route is based on the average number of daily passengers per route, given

in the table below.

Route	A	B	C	D	E	F
Average # of passengers	4,360	5,130	7,080	10,245	15,535	22,650
Quota						

II. Hill Method

Geometric Mean:

Example: If the quota is 11.7, find the geometric mean:

Steps:

1. Round quotas based on the geometric mean. Add the resulting integers together. If the total equals the number of items to be apportioned, stop. If not, go on.
2. Divide each population by a modified divisor (d).
3. Round the resulting numbers based on the geometric mean. Add the resulting integers together.
4. If the total equals the number of items to be apportioned, stop. If not, repeat steps 2 and 3.

1. Discrete Town has 5 districts. District 1 has 3640 people. District 2 has 5760, District 3 has 10,300 people, District 4 2,500 and District 5 has 7,800. The town council has 30 seats.

District	1	2	3	4	5
Population	3,640	5,760	10,300	2,500	7,800
Quota					

2. An HMO has 70 doctors to be apportioned among six clinics. The HMO decides to apportion the doctors based on the average weekly patient load for each clinic

Clinic	A	B	C	D	E	F
Average Weekly Patient Load	309	591	389	683	419	479
Quota						

The republic of Mathtown is composed of four states, A, B, C, and D. According to the country's constitution, the congress will have 30 seats, divided among the 4 states according to their respective populations

State	A	B	C	D
Population (in thousands)	275	383	465	767
Quota				