Calculus

Calculus is divided into 2 main parts:
a) Differential Calculus
b) Integral Calculus

Differential Calculus: The study of rates of change within a function. The idea of differential calc is to determine whether functions are increasing or decreasing and how fast they are doing so.

Rates of change are used in a lot of different areas of study.

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Physics: Newtons law of cooling - the rate of change of the temp of a body with respect to time is proportional to the difference between the temp of the body and that of the surroundings.

Biology: Under ideal growth conditions, the rate of growth of a colony of bacteria is proportional to the number of bacteria at that time.

Business: As the number of items being produced increases, the cost of producing each item decreases, but the rate of decrease diminishes.

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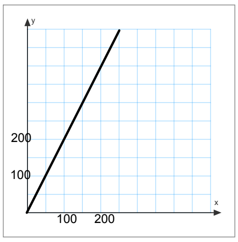
Rates of Change:
I. Linear Functions

The Mississippi River is about 2340 miles long, and it flows at an average rate of about 2 miles per hour. The graph relating distance of a raft vs. time on the river is graphed below:



How would you find the rate of change of the line?

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II. Non-Linear Functions

All through the year the length of a day from sunrise to sunset changes. (unless you live on the equator). Below is a table of the length of days for places that are at 50° latitude. (Vancouver Canada, Frankfurt Germany, Prague Czech Republic.)

Length of day in minutes on the 1st day of the month

Date	Jan 1	Feb 1	March 1	April 1	May 1	June 1	July 1	Aug 1	Sept 1	Oct 1	Nov 1	Dec1
day of year	1	32	60	91	121	152	182	213	244	274	305	335
length	490	560	658	775	882	964	977	914	809	699	583	504

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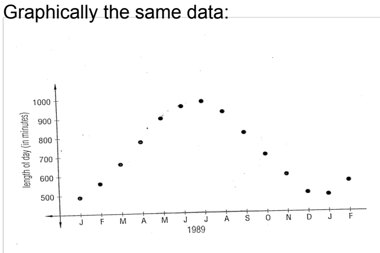
Graphically the same data:



How would you find the rate of change for this non-linear function?

Will it be constant?

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Date	Jan 1	Feb 1	March 1	April 1	May 1	June 1	July 1	Aug 1	Sept 1	Oct 1	Nov 1	Dec1
day of year	1	32	60	91	121	152	182	213	244	274	305	335
length	490	560	658	775	882	964	977	914	809	699	583	504

For linear we found the slope. Look at different sets of points and find the slope. Since the length of days does not change evenly throughout the year, when finding the slope we are finding the "average rate of change" over that time period.

A. Average rate of change of a function from x_1 to x_2 is the slope through the line $(x_1, f(x_1))$ and $(x_2, f(x_2))$.
or

$$\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

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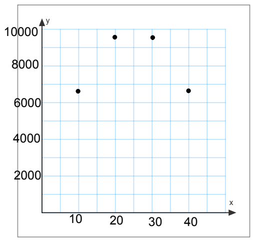
Ex. 1: A projectile is propelled into the air with an initial velocity of 800 feet per second. If only the effect of the Earth's gravity is considered, its height in feet after t seconds is given by the function $h(t)=800t-16t^2$. The graph below shows the function over the interval $0 \le t \le 40$. Find the average rate of change over the following intervals:

a. $10 \le t \le 20$ b. $20 \le t \le 30$ c. $30 \le t \le 40$



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Our average rate of change formula for all of our examples has been the same: $\frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{f(x_2) - f(x_1)}{x_2 - x_1}$

There is a 4th formula that is widely used in calculus. It is a formula that is in terms of *f, x and* Δx.

Since $\Delta x = x_2 - x_1$, then $x_2 = x_1 + \Delta x$. Then by substitution,

$$\frac{f(x_1 + \Delta x) - f(x_1)}{\Delta x}$$

This is called the difference equation of f over the interval from x_1 to $(x_1 - \Delta x)$

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ex. 2

$h(t) = 800t - 16t^2$

Find the formula for the difference quotient giving the average rate of change of h for each interval.

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ex. 3 Use the formula from example 2 to find the projectile's average velocity from t to $t+\Delta t$ for t = 5 and:

a. $\Delta t = 1$ b. $\Delta t = .5$ c. $\Delta t = -.1$

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Lesson 2: The Derivative at a Point

Quick Review - By looking at the motion of a projectile modeled by the function $h(t) = 800t - 16t^2$.

-the velocity of the projectile was different at each moment.

- We found the average velocity over time intervals

-We also started to find the velocity at any given point of time How did we do this?

What happened when we used our formula and chose smaller and smaller values for Δt?

-What we were really finding was the instantaneous velocity

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I. Instantaneous Velocity

A. Def: Instantaneous Velocity = $\lim_{\Delta t \to 0} \frac{f(t + \Delta t) - f(t)}{\Delta t}$

B. Examples

1. $h(t) = 800t - 16t^2$

Yesterday, $\frac{h(t + \Delta t) - h(t)}{\Delta t} = 800 - 32t - 16\Delta t$

problem: Find the slope (instantaneous velocity) at t= 5 sec.

Now, $\lim_{\Delta t \to 0} 800 - 32t - 16\Delta t$

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ex2. $f(x) = x^2 + 4$. Find the instantaneous rate of change of f at x = 3.

Answer: $\lim_{\Delta t \to 0} \frac{f(t + \Delta t) - f(t)}{\Delta t}$

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Connect to Geometry: When we found the rate of change between 2 points, we really found the slope of the secant line through those 2 points.

Now that we are finding the instantaneous rate of change we are finding the slope of the tangent line at a point.

This happens in so many places that it has been given a specific name: Derivative

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II. Derivatives

A. Def: The derivative of a function *f* at *x* is denoted *f'(x)*, is given by:

$$\lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

provided the limit exists and is finite.

B. Requirement:
To have a derivative a function must be:
a) continuous
b) Must be "smooth" at the point you are finding the derivative of.

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C. Examples
A spherical balloon is being blown up by the function: $V(r) = \frac{4}{3}\pi r^3$
What is changing or varying?

a) Compute V'(1), the instantaneous rate of change of V at r=1

b) Compute V'(2)

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Warm Up:

Find the derivative of $f(x)= 2x^2 + 3x$ at 4.

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Extra Practice: FInd the derivative of

$f(x) = 3x^2 + 6x - 4$ at a time of 4 sec.

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Derivative Short Cut:

Exponent times coefficient, then lower the coef by 1.

1. $5x^2 - 8x + 12$
2. $12x^2 + 14x^3 - 24$
3. $58 - 16x^8$
4. Find the derivative at 5 for $24x^3 - 12x + 9$
5. FInd the derivative at 12 of f(x) = 12

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First derivative is velocity

second derivative is acceleration.

$f(x) = -16x^2 + 48x + 12$

Find the velocity at 1.5 sec and the acceleration at 2 sec.

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Warm Up

A projectile has a height of $h(t) = -16t^2 + 70t$.

a) What is the height at 3 sec?

b) What is the maximum height?

c) What is teh instantaneous velocity at 3 sec?

d) What is the velocity at the max height?

e) What is the initial rate of change?

f) When does it his the ground?

g) What is the velocity when it hits the ground?

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Comparing the graphs of a funtion to the graph of the derivative:

Original f(x)	Derivative f'(x)
Increasing	positive (above x-axis)
Decreasing	negative (below x-axis

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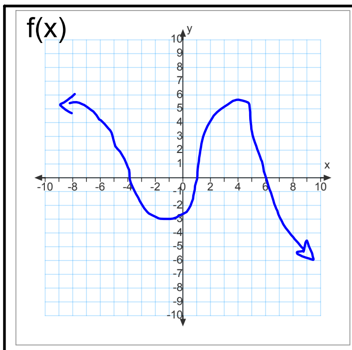

1. On what intervals is f'(x) positive?
2. negative?
3. f'(x) = 0?

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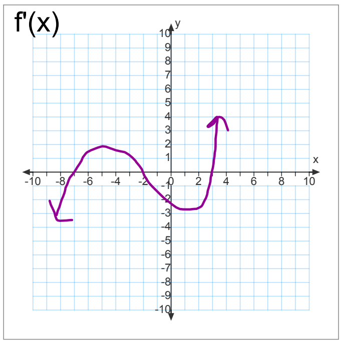

1. Where is f(x) increasing?
2. decreasing?
3. Where does f(x) have a relative max or min?

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1. Given the function: $1/3\, x^3 + 4x^2 + 15x$, use the derivative to find the intervals of decreasing and increasing. Also find the max and mins.

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2. If you have 60 feet of fencing, what is the largest area of a rectangle you can create?

3. 210 feet of fencing?

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